Modelling Maximal Oxygen Uptake in Athletes: Allometric Scaling Versus Ratio-Scaling in Relation to Body Mass

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Introduction

Aerobic fitness data of Singaporean athletes are relatively scarce. Yet these data are important as aerobic fitness forms the basis of all training programmes to enhance performance during competitions. The “gold standard” of aerobic fitness is maximal oxygen uptake or \( \dot{V}_O_2 \) peak. There is considerable debate about how best to normalise maximal oxygen uptake or \( \dot{V}_O_2 \) peak for differences in body size in children and in adults. By convention, \( \dot{V}_O_2 \) peak is expressed as a ratio standard (i.e., mL/kg BM\(^{1.0}/min\)); and by expressing \( \dot{V}_O_2 \) peak in this manner; it is assumed that the influence of body mass (BM) or body size is removed. In spite of the theoretical and statistical limitations of the ratio standard, the use of the ratio standard in normalising \( \dot{V}_O_2 \) peak remains endemic. There is strong persuasion that allometric or power function modelling may be theoretically, physiologically and statistically superior to alternative scaling methods, but not all researchers are in agreement.\(^1\) An oft-repeated argument among exercise scientists is that the traditional ratio standard of \( \dot{V}_O_2 \) peak (mL/kg BM\(^{1.0}/min\)) fails to render \( \dot{V}_O_2 \) peak independent of BM. Therefore, the most appropriate way to remove the confounding effects of BM, so as to facilitate comparisons in exercise performance across groups of different body sizes, is to adjust \( \dot{V}_O_2 \) peak using the power function relationship: \( \dot{V}_O_2 \) peak = \( aBM^b \), where \( a \) is called the scaling constant and \( b \) is called the scaling exponent.\(^2\)

In contrast, some researchers\(^1\) have criticised the indiscriminate use of allometric modelling without first verifying that the assumptions for its use in scaling the data sets are valid and defensible. These assumptions include: (i) there is a curvilinear relationship between \( \dot{V}_O_2 \) peak and the body size descriptor [i.e., in most cases it is BM or fat-free mass, (FFM)], (ii) in log-transformed allometric model, there is a strong and linear relationship between the dependent variable (\( \dot{V}_O_2 \) peak) and the independent variable (i.e., body size descriptor e.g. BM), (iii) that \( \dot{V}_O_2 \) peak and
body size are heteroscedastic, and (iv) the adjustment of \( \dot{V}_O_2 \) peak using power function modelling (i.e., \( \dot{V}_O_2 \) peak (L/min)/BM) is indeed body size independent. In most cases of allometric scaling in exercise science, these recommendations for the appropriate use of allometric scaling are rarely heeded and should therefore be adequately addressed. This is important since it appears that allometric scaling is becoming the method of choice for partitioning out the effects of body size on physiological or human performance data sets. Indeed, Winter called for the "preferential use of allometric modelling" and Vanderburg and Katch encouraged future research to apply allometric scaling to many physiological, human performance and anthropometric variables.

A related issue is the recommendation by researchers to derive sample-specific allometric scaling factors that describe exactly the relationship between the dependent (performance variable) and independent variable (body size descriptor) to appropriately adjust for the influence of size rather than to use theoretical exponents that are predicted by dimensionality theory. The use of the mass or size exponents that are based on dimensional theory or biological similitude may or may not be applicable to the sample being examined due to differences in age, fitness levels, homogeneity or heterogeneity of the sample.

The primary aims of the present study were (a) to verify the validity of using allometric scaling to adjust \( \dot{V}_O_2 \) data peak in male and female athletes for differences in body size, (b) to compare the \( \dot{V}_O_2 \) peak of male and female athletes using power function or allometric modelling and (c) to evaluate the success of the allometric model in providing a dimensionless physiological variable (i.e., \( \dot{V}_O_2 \) peak (L/min)/BM), an index that is unrelated to BM.

**Materials and Methods**

**Subjects**

One hundred fifty-eight males and 28 females were tested in the study. These subjects were considered as junior and senior level athletes representing the country in different sports (e.g., soccer, hockey, runners, etc). For male subjects, 13% of them were endurance athletes and the rest were game athletes. For female subjects, 8% of them were endurance athletes and the rest were game athletes. Preliminary analysis using dependent t-tests showed that \( \dot{V}_O_2 \) peak (L/min) was not significantly different \((P>0.05)\) between endurance runners and game athletes in male and female athletes. Consequently, the data in male and female subjects were pooled. The data were collected from athletes who had visited the exercise physiology laboratory of the Sports Medicine and Research Centre’s exercise physiology laboratory over the last decade. The benefits and risks of the tests were communicated to all the athletes and all signed a written informed consent prior to testing. For athletes who were tested several times during the period, the test in which his or her highest maximal oxygen uptake value was recorded and was subsequently used in the analysis.

**Anthropometric Measurements**

The anthropometric measurements were made using calibrated scales. For stature, a Holtain stadiometer was used, and for BM, a weighing scale (Seca, Hamburg, Germany) was used prior to the tests. Stature and BM are common body size descriptors that are used to normalise exercise performance.

**Treadmill Test**

The maximal \( \dot{V}_O_2 \) test for the athletes was conducted using a continuous and an incremental protocol on a treadmill (Marquette 1900, Milwaukee, WI), following familiarisation with treadmill running. Athletes performed a standardised warm-up consisting of 2 minutes of walking on the treadmill, followed by a trial to determine a self-selected running speed that was used as the starting speed for the incremental maximal oxygen uptake test. The athletes were instructed that the running speed should be one that they could endure for 12 to 15 minutes. Once chosen, the athlete ran at this speed for at least 2 minutes to allow steady state to be achieved. This was followed by 10 minutes of stretching exercises for the lower limbs before the commencement of the treadmill test. All athletes were instructed to give a maximal effort and were verbally encouraged to run till volitional exhaustion was achieved.

The test commenced with a treadmill velocity that was between 8.0 and 14 km/h with a zero percent grade for the first 2 minutes. Then for the next 5 minutes, treadmill elevation was increased by 2% at the end of each minute. Subsequently, the elevation was increased by 1% every minute until the subject achieved volitional exhaustion.

Respiratory variables for oxygen uptake (\( \dot{V}_O_2 \)) and carbon dioxide (\( \dot{V}_C_O_2 \)), minute ventilation (\( \dot{V}_E \)) and respiratory exchange ratio (RER) were recorded every 20 seconds using an open-circuit spirometry system (Sensormedics 2900Z, Yorba Linda, CA). The oxygen and carbon dioxide gas analysers were calibrated prior to each test with known concentrations of standard gases and the flowmeter was calibrated using a 3 L syringe. Heart rate (HR) data were monitored throughout the test by means of short-range radio telemetry (Sport-tester, Polar Electro Oy, Finland).

**Criteria for Determination of Maximal \( \dot{V}_O_2 \) Effort**

All respiratory variables at maximal \( \dot{V}_O_2 \) were reported as the highest 60 s (i.e., highest of three 20 s consecutive
readings). The subject’s maximal $V\dot{O}_2$ was achieved when 2 of the criteria were achieved: i) RER >1.05, ii) HR at test termination of >95% of age-predicted HR_{max} (based on 220 minus age in years) and (iii) exercising to volitional exhaustion.

**Statistical Analyses**

All data were analysed using SPSS for Windows version 11.5. Descriptive statistics [mean ± standard deviation (SD)] were generated for anthropometric characteristics of the participants (i.e., age, BM and stature) and all variants of the dependent variable (i.e., $V\dot{O}_2$ peak).

Sex differences criteria measurements were analysed using one-way analysis of variance (OW-ANOVA).

A regression technique applied on log-transformed data (i.e., Ln peak $V\dot{O}_2$ in L/min and Ln body mass, BM in kg, and Ln stature) provided the allometric parameters $b$, respectively for male and female athletes. Results showed that Ln BM rather than Ln stature was the dominant predictor for Ln peak $V\dot{O}_2$ and Ln BM was used in subsequent analyses. The natural log-transformation of the presumed allometric relationship between peak $V\dot{O}_2$ and BM is given by: Ln peak $V\dot{O}_2$, (L/min) = Ln A + b Ln BM, where Ln A is the natural log-transformed scaling constant, corresponding to the Y-intercept and $b$ is the scaling factor.\(^3\) Subsequently, the derived equations for male and female athletes were written as (Y i.e., $V\dot{O}_2$ peak) = A.BM\(_{specific}\)\(^b\) exponent. Statistical checks for normality of the data sets, homogeneity of variance, and residual analyses of the natural log-transformed data sets after regression analyses to derive the $b$ exponents were also conducted. This was to ensure that the underlying assumptions for the use of allometric scaling were met. The ability of the allometric model to provide a mass-free variable was also evaluated by examining the relationship between the power function ratio (i.e., $V\dot{O}_2$ peak/BM\(^b\)) and BM using Pearson Product Moment correlation. The level of statistical significance was accepted as $P <0.05$.

**Results**

**Normality of Distribution and Heterogeneity/Homogeneity of Variance**

Statistical analyses demonstrated that there was normality of distribution for peak $V\dot{O}_2$ in L/min for both male and female athletes (Shapiro-Wilk statistic, $P >0.05$) but a visual inspection of Figure 1 shows heterogeneity of $V\dot{O}_2$ peak in relation to BM (spread of $V\dot{O}_2$ peak scores increased with BM) especially in the male athletes, but the difference in variance was not significantly different between male and female athletes (Levene test statistic, $P >0.05$).

**Physical Characteristics and Physiological Responses at Peak $V\dot{O}_2$**

The physical characteristics and the physiological responses at $V\dot{O}_2$ peak are summarised in Table 1. Male athletes were taller, had higher BM and ventilation rates at $V\dot{O}_2$ peak than female athletes, but there was no sex difference in maximal exercise HR and RER at $V\dot{O}_2$ peak.

**Allometric Scaling of Peak $V\dot{O}_2$**

Figure 2 shows the log-linear relationships between $V\dot{O}_2$ peak and body mass in male (N = 158) and female (N = 28) athletes, where heteroscedasticity of the data is demonstrated (i.e. the variance in $V\dot{O}_2$ peak increased with BM). The resultant sex-specific equations for the non-linear fitted model were: for male athletes, $V\dot{O}_2$ peak = 2.23BM\(^{0.67}\) and for female athletes, $V\dot{O}_2$ peak = 2.23BM\(^{0.24}\).

![Figure 1](image_url)  
*Denotes significant sex difference at $P <0.05$. $V\dot{O}_2$ peak is maximal oxygen uptake.

### Table 1. Physical Characteristics and Physiological Responses at $V\dot{O}_2$ peak in Male and Female Athletes

<table>
<thead>
<tr>
<th>Variable</th>
<th>Male athletes (n = 158)</th>
<th>Female athletes (n = 28)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (y)</td>
<td>21.7 ± 4.9</td>
<td>21.9 ± 7.0</td>
</tr>
<tr>
<td>Body mass (kg)</td>
<td>64.8 ± 8.6</td>
<td>53.0 ± 7.0*</td>
</tr>
<tr>
<td>Stature (m)</td>
<td>1.72 ± 0.06</td>
<td>1.61 ± 0.07*</td>
</tr>
<tr>
<td>Minute ventilation at $V\dot{O}_2$ peak (L/min)</td>
<td>123.7 ± 19.5</td>
<td>87.7 ± 10.6*</td>
</tr>
<tr>
<td>Heart rate at $V\dot{O}_2$ peak (beats/min)</td>
<td>190 ± 9</td>
<td>189 ± 9</td>
</tr>
<tr>
<td>Respiratory exchange ratio at $V\dot{O}_2$ peak</td>
<td>1.11 ± 0.05</td>
<td>1.10 ± 0.04</td>
</tr>
</tbody>
</table>

* Denotes significant sex difference at $P <0.05$.
transformed peak $\text{VO}_2$ and log-transformed BM in male and female athletes. As there was a significant difference in the Sex*Ln BM interaction ($F = 48.19$, $P < 0.05$), separate $b$ exponents were derived for the male ($b = 0.67$; 95% confidence interval (CI), 0.57-0.81) and female ($b = 0.24$; 95% CI, 0.09-0.56) athletes, respectively.

The 95% CI for the derived sex-specific $b$ exponents excluded 1.0. This means that for both male and female athletes, $\text{VO}_2$ peak (i.e., dependent variable) increased faster than the increase in BM (i.e., independent variable), and that $\text{VO}_2$ peak increased faster in female athletes than male athletes, expressed in relation to BM.

From Figure 1, the resultant sex-specific equations for the fitted non-linear model were:

- $\text{VO}_2$ peak = 2.23BM$^{0.67}$ for male athletes
- $\text{VO}_2$ peak = 2.23BM$^{0.24}$ for female athletes
The adequacy and appropriateness of allometric scaling of the data to produce a BM-independent variable was demonstrated by the examination of residual analyses. Figure 3 indicates that the residuals were normally distributed and Figure 4 shows that assumptions of linearity and homogeneity of variance of the log-transformed dependent variable (i.e., \( \ln V_{O2} \) peak) were met. Ln is natural log.

Moreover, there were no significant correlations \( (P > 0.05) \) between male- \((V_{O2} \text{ peak}/BM^{0.67})\) and female- \((V_{O2} \text{ peak}/BM^{0.24})\) derived power function ratios and BM. This is depicted in Figure 6.

This result is in contrast to the result shown in Figure 5, where the use of the traditional ratio-scaling resulted in significant negative correlations with BM in male and female athletes; i.e., peak \( V_{O2} \) in mL/kg BM\(^{1.0}/\text{min} \) was still significantly related to BM.

**Sex Difference in Peak \( V_{O2} \)**

Figure 7 provides a summary of sex differences in \( V_{O2} \) peak in absolute terms, when ratio-scaled to BM\(^{1.0}\) and when allometrically-scaled to BM\(^{b}\) using the sex-specific derived \( b \) exponents (i.e., \( b = 0.67 \) for male and \( b = 0.24 \) for female athletes). In female athletes, \( V_{O2} \) peak in L/min was 67.8% that of the male athletes, \( V_{O2} \) peak in mL/kg BM\(^{1.0}/\text{min} \) was 83.4% of the male athletes. Given that the allometrically derived \( b \) exponents for male \( (b = 0.67) \) and...
female \((b = 0.24)\) athletes were distinctly different, direct sex comparison of the power function ratio was precluded. However, Figure 6 depicts that the male and female athletes had distinctly different BM relationships with \(\text{VO}_2\) peak.

**Discussion**

The primary aim of the study was to describe the maximal oxygen uptake or \(\text{VO}_2\) peak of the male and female athletes using ratio-scaling and allometric scaling or power function ratios.

**Appropriate Use of Allometric Modelling**

A visual plot of the \(\text{VO}_2\) peak data of male and female athletes in the present study (Fig. 1) the relationship between \(\text{VO}_2\) peak and BM was curvilinear and following allometric modelling, separate equations were derived for male and female athletes (Fig. 2). The appropriateness of using power function modelling or allometric scaling was reinforced by the results of log-linear regression analyses by an examination of the resultant residuals (Figs. 3 and 4).

In essence, the regression diagnostics showed that the assumptions (i.e., normality of distribution of the residuals, linearity and homogeneity of the residuals, indicating a constant error variance) advocated by Winter\(^5\) and Welsman and Armstrong\(^3\) for the appropriate use of allometric modelling of exercise data, were adhered to.

Importantly, a follow-up examination of the relationship between the derived power function ratio and BM revealed no significant correlations, thereby confirming that the computed power function ratios (i.e., peak \(\text{VO}_2 / \text{BM}^{0.67}\) for male athletes [i.e., \(0.22 \pm 0.03 \text{ mL/kg BM}^{0.67}/\text{min}\)] and peak \(\text{VO}_2 / \text{BM}^{0.24}\) for female athletes [\(0.98 \pm 0.10 \text{ mL/kg BM}^{0.24}/\text{min}\)] were indeed mass-independent (Fig. 6). These results are in contrast to the data shown in Figure 5, where the traditional ratio-scaling of peak \(\text{VO}_2\) in mL/kg BM/min was still significantly correlated with BM; i.e., it was not mass-free as it was purported to be.

**Sex-specific Mass Exponents for \(\text{VO}_2\) Peak and Sex Differences in \(\text{VO}_2\) Peak**

Results of allometric modelling revealed mass exponents of \(b = 0.67\) and \(b = 0.24\) for male and female athletes in relation to \(\text{VO}_2\) peak. There are few studies that report exclusive mass exponents for female athletes but Berg et al.\(^1^1\) reported mass exponents in 7 different groups of adults that ranged between \(b = 0.47\) and \(b = 0.86\), with a weighted mass exponent of \(b = 0.71\). Others\(^4,1^2\) reported mean mass exponents of \(b = 1.02\) and \(b = 1.0\) in relation to \(\text{VO}_2\) peak in adults, albeit from heterogeneous populations (i.e., wide age range, different fitness levels, etc).

Heil\(^1^3\) argued that for samples that are homogeneous in nature (i.e., similar training background, age and body height), a mass exponent of \(b = 0.67\) is appropriate, an argument that is affirmed by the result of male athletes in the present study. In female athletes, the mass exponent (i.e., \(b = 0.24\)) was markedly different from that of the male athletes, thereby precluding the use of a common \(b\) exponent in the computation of common power function ratios. Instead, the power function ratios had to be computed separately, using the sex-specific mass exponents. However, statistical checks revealed that allometric scaling was appropriate since there were no significant correlations between the sex-specific power function ratio and BM (Fig. 6), demonstrating that the power function ratios were indeed BM independent.\(^3\)

The lower mass exponents of the female athletes could be speculatively due to (i) the difference in body composition between male and female athletes, (ii) the relatively smaller sample size of female subjects compared to male subjects, or (iii) Male and female athletes had different \(b\) exponents for \(\text{VO}_2\) peak. For example, one interpretation from the
present result of a lower mass exponent in female athletes was that \( \dot{V}O_2 \) peak increased at a relatively faster rate than the increase in BM compared to the situation in male athletes. This finding is unique and the physiological explanations are not readily apparent but sample-specificity in the results cannot also be discounted.

In terms of sex difference in \( \dot{V}O_2 \) peak, female athletes attained 66.9% that of male athletes when expressed in L/min and 83.4% that of male athletes when it was expressed in mL/kg BM\(^{0.67}\)/min (Fig. 7). This result is in close accord to the results of elite male and female endurance athletes reported by Docherty et al.\(^1\) The corresponding female-to-male ratios (x100) were in absolute terms (3.52 vs. 5.26 L/min; 66.9 %) and in BM ratio-scaled terms (66.1 vs. 79.1 mL/kg BM/min; 83.6 %), respectively.

In the cited study, which also employed allometric scaling techniques to normalise male and female \( \dot{V}O_2 \) peak data of adults, a common mass exponent of \( b = 0.67 \) were used, but was not empirically derived from the study sample.\(^{14}\) The authors reported that \( \dot{V}O_2 \) peak, expressed as a power function ratio (i.e., peak \( \dot{V}O_2 /BM^{0.67} \)) was significantly greater than those of female adults. Though no direct sex comparisons in allometrically scaled \( \dot{V}O_2 \) peak could be made in the present study, due to the dissimilar mass exponents, the present result resonates with the view that different numerical values for the mass exponents could result in different biological and theoretical explanations.\(^{15}\) Future studies should explore using FFM as the body size descriptor, which was unavailable in the present study, instead of BM since several studies have shown that FFM as the best single predictor of \( \dot{V}O_2 \) peak.\(^{6,16}\)

In summary, the present data showed that ratio-scaling of \( \dot{V}O_2 \) peak to BM in trained athletes did not create a size-free variable, and should not therefore be used indiscriminately because of convenience to compare between groups of male and female athletes. Data from the present study showed that when ratio-standards were used, \( \dot{V}O_2 \) peak was still affected by body size. Instead, \( \dot{V}O_2 \) peak, that was independent of the effect of BM in male and female athletes, was best described using 2 separate and allometrically-derived sex-specific regression equations.

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REFERENCES